point $r(y)$ must approximate a Rankine vortex. This, in fact, is implied in Eq. (3), where there is no longer a "warning signal."

To illustrate, Fig. 1 presents two numerical results derived from Eq. (3). They correspond to the two distributions†

$$\gamma_y(y) = 1 + 1.5y^2 - 2.5y^4 \tag{5a}$$

$$\gamma_y(y) = 1 - y^2 \tag{5b}$$

which are shown as distributions (a) and (b) in Fig. 1. Distribution (a) has a slight central dip and is chosen such that $r_c = y_c = 1$; the separation distance between the two trailing vortices just equals the wing span. With a deeper dip, the separation distance would be even larger. In contrast, for distribution (b), one finds $r_c = y_c = 2/3$, smaller than for the elliptic lift distribution.[3]

The induced drag of both (a) and (b) obeys the formula

$$C_{Di} = (9/8)C_L{}^2/\pi A \tag{6}$$

The factor (9/8) in Eq. (6) is replaced by 1 in the case of minimum induced drag (the elliptic lift distribution).

For each distribution γ_y, Fig. 1 shows the distribution $v_T(r)$ of the tangential speed in the core.‡ On the v_T-curves, the points y on the wing are noted where the local changes of $\gamma_r(r)$ in the core are shed. The structure of the vortex core depends considerably upon the lift distribution. Distribution (a), having the larger total lift, has to have the larger energy content according to Eq. (6). This is reflected in the larger core speeds v_T. Outside their cores, the two vortices are identical with the same Rankine vortex.

Distribution (a) has a horizontal tangent at about $y = 0.55$. As discussed above, this does not produce any irregularity in the corresponding v_T curve. This sample result suggests that the mere occurrence of a zero in γ_y' should not be sufficient to invalidate Eq. (3). On the other hand, if the dip is so deep that the lift changes sign, then, according to Eq. (3), $r(y)$ goes to infinity and changes sign. This analytical result is physically unacceptable. Before this happens, there is a range of uncertainty. It must be left to the experiment to establish the criterion to be used.

We close with a remark on stability analyses. In the experiments of Bilanin and Widnall,[6] the spanwise lift distribution was varied periodically by means of flaps, somewhat in the manner of Fig. 1, but such that the total lift remained about constant (see Fig. 3 of Ref. 6). According to Eq. (3), the relation between y and r remained about invariant in the vicinity of the tip, which corresponds to the region near the inner core. Consequently, since the circulation shed near the tips was strongest at the times when the outer flaps were down, the circulation next to the inner core (the essential agent which produces self-induction) was *strongest* at the corresponding positions along the trailing vortices. At the same positions, the Rankine vortices were at their *weakest*. In the analysis,[6] no allowance was made for the outer cores; the Rankine vortices were assumed to reach to the inner cores. One sees from this example that Eq. (3) might open the way to a more realistic stability analysis.

†Neither distribution fulfills the tip condition $\gamma_y'(1) = -\infty$ (which is immaterial for this illustration).

‡From Eq. (5b), Eq. (3) yields $r = (2 - y - y^2)/3(1 + y)$. The reader may compare this result with the much more complicated form of Brown[3] Eq. (9) to illustrate the simplification which arises from Eq. (3).

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Design of Crashworthy Aircraft Cabins Based on Dynamic Buckling

Cary A. Fisher* and Charles W. Bert†

University of Oklahoma, Norman, Okla.

Nomenclature

A = stiffener cross-sectional area (in.^{2})
E = Young's modulus (psi)
$\bar{F}_{cr}$ = nondimensional buckling load, $\bar{F}_{cr} = \hat{N}_{cr}R/Eh^2$
h = unstiffened shell wall thickness (in.)
I_0 = moment of inertia of stiffener about shell middle surface (in.^{4})
J = polar moment of inertia of stiffener about its centroid (in.^{4})
l,L = ring center-to-center spacing and shell length (in.)
M,n = number of stringers and number of circumferential waves
$\hat{N}_{cr}$ = externally applied axial-load resultant for dynamic buckling (positive in comp)
r,s = subscript referring to ring and stringer quantities
R = radius of cylinder middle surface (in.)
$\bar{z}$ = distance from centroid of stiffener to shell middle surface (in.)

Introduction

ANALYTICAL or experimental investigations of the structural integrity of light aircraft cabins under crash impact conditions have been lacking. Light aircraft have design loadings much lower than those of military or transport design and thus have fewer stiffeners, i.e., much wider stiffener spacing. Therefore, the popular analytical approach of considering stiffeners to be "smeared" over the shell surface[1] is not valid for light aircraft and one should account for the discreteness of the stiffeners, as has

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*Capt., US Air Force, on leave of absence from Department of Engineering Mechanics, U.S. Air Force Academy. Associate Member AIAA.

†Professor and Director, School of Aerospace, Mechanical and Nuclear Engineering. Associate Fellow AIAA.

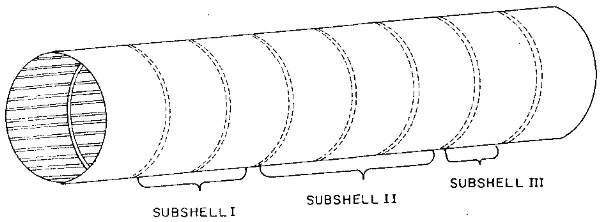

Fig. 1. General aircraft cabin structure.

been done in only a few analyses of free vibration and static buckling.[2,3]

Previous theoretical and experimental investigations[4-6] of the buckling of cylindrical shells subjected to a rapidly applied axial load have been limited to unstiffened shells, which would be expected to have quite different behavior than that of stiffened shells. Recently, however, a nonlinear analysis was developed to treat dynamic buckling of a circular cylindrical shell having discrete rings and stringers and subjected to a rapidly applied axial compressive load.[7] This analysis is applied here to an idealized stiffened-cylinder model representative of current-design light-aircraft cabins. Various parameters relating to the stiffeners are varied parametrically to elucidate design concepts for increasing cabin crashworthiness based on the criterion of dynamic buckling load.

Representative Aircraft Cabin Structure

Because of the large number of geometric and physical parameters involved, it is impractical to present general results. However, some computed results for stiffened cylinders representative of light-aircraft fuselage structures are presented. To determine the geometric parameters of a typical light-aircraft fuselage structure, four light aircraft companies were contacted and Table 1 is based on the data they provided. All rings and stringers are idealized to be uniform, of constant thickness, internally located, and evenly spaced. No provision is made for cutouts for doors, windows, etc. Table 1 shows that all four aircraft have approximately the same stiffener area, although some of the other stiffener quantities vary by as much as 50% due to different geometric shapes of stiffeners used. For example, the stringers in aircraft C are channel-sections, while those in aircraft D are L-sections. Based on the data, a representative stiffened cylinder having the parameters listed at the bottom of the table was chosen for investigation.

Because of the relatively wide spacing between the rings on light aircraft, local buckling of the shell between the rings was the expected mode of the failure, rather than general instability. Local buckling between rings is defined as the buckling mode in which the rings have little or no radial deformation and the cylinder buckles between the rings. General instability is the buckling mode in which the rings deform radially and the cylinder wall and rings buckle as a composite wall. The analysis used can handle either type of instability.

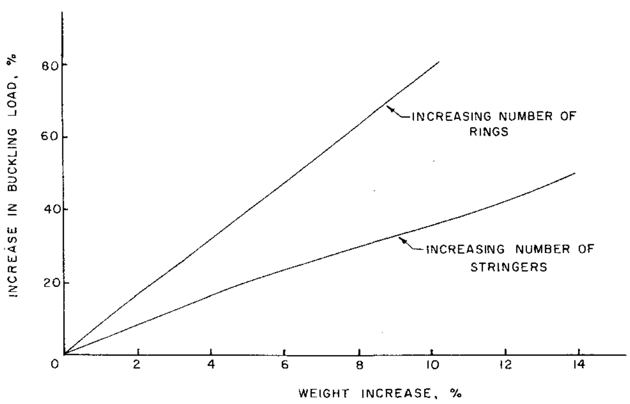

Fig. 2. Minimum-weight design considerations.

Modeling Using Subshells

An aircraft cabin consists of a long nearly cylindrical section with a number of repeating bays, so that one would expect that buckling of the entire cabin can be studied by considering a representative smaller portion or subshell of it. Then fewer assumed-mode terms in the deflection function would be sufficient to adequately model the dynamic behavior. To validate this hypothesis, the long cylinder shown in Fig. 1 was divided into three subshells:‡ SS (subshell) I had one ring and $L/R = 2$, SS II had two rings and $L/R = 3$, and SS III had no rings and $L/R = 1$. A buckling analysis was performed on SS I and II, with results as tabulated in Table 1. The lowest buckling loads for SS I and II were approximately the same and occurred for the same n, indicating the similarity of buckling shapes. As expected, SS II required more terms (5) than SSI (3 terms) for convergence.

Subshell III was used as a further check on the convergence of the SS I and II results and as a demonstration of local buckling of the subshell. When local buckling occurs between the rings, the rings can exert only a torsional restraint. Thus, a lower bound for buckling load is obtained by considering the stringer stiffened portion between the rings (SS III in Fig. 1); the corresponding buckling load is listed in the tabulation above. The closeness of the SS I and II values to this lower-bound value demonstrates that local buckling is the buckling mode for the representative cylinder.

Local buckling between the rings is the dominant instability mode, since local buckling between the stringers occurs at higher load. Because of the great increase in computer running time and core space for each additional assumed-mode term, the number of circumferential assumed-mode terms is limited to one. The number of axial terms is increased until convergence is obtained.

Stiffener Design Considerations

The influence of the number of stringers (M) on the dynamic buckling load of SS II is shown in Table 2. All shell and stiffener parameters were held constant except M. If the number of stringers of the aircraft C type were doubled, the minimum dynamic buckling load would be increased by over 50%. If $M<30$, local buckling between the stringers would probably be the dominant mode of buckling, with a corresponding decrease in buckling load. This suggests that a minimum-weight design would be one having the number of stringers just sufficient to prevent local buckling between the stringers. The influence of the spacing on the buckling load is shown in Table 3.

If the distance between rings is cut in half, the buckling load is increased by about 50%. For the representative cylinder considered, it is more efficient weightwise to increase the number of rings, rather than stringers, if increased dynamic buckling load is desired (Fig. 2). For example, for a weight penalty of only 10%, addition of rings increases the buckling load by 78%, while addition of stringers gives an increase of only 35%.

The effects of varying the ring and stringer cross-sectional areas and eccentricities were studied. No significant change in buckling load was found for increases of 25 to

‡Even the shortest subshells treated here had a Batdorf length parameter of 190, which far exceeds the value above which the boundary conditions are relatively unimportant.[8]

Table 1 Geometric characteristics of four light-aircraft cabins and one representative of all four

Aircraft	Type	Cabin length, in.	R, in.	h (in.)	M	A_s (in.^{2})	I_{os} (in.^{4})	J_s (in.^{4})	$\bar{z}_s$ (in.)	A_r (in.^{2})	I_{or} (in.^{4})	J_r (in.^{4})	$\bar{z}_r$ (in.)
A	Single engine	112	25	0.030	33	0.032	0.006	0.0049	0.038	0.094	0.088	0.039	0.763
B	Single engine	110	25	0.030	22	0.065	0.015	0.0059	0.466	0.142	0.320	0.120	1.210
C	Twin engine	168	34	0.030	27	0.046	0.013	0.0078	0.413	0.165	0.264	0.102	1.013
D	Twin engine	127	31	0.040	30	0.035	0.003	0.0022	0.187	0.133	0.461	0.159	1.513
Representative	Various		32.0	0.040	30	0.046	0.013	0.0078	0.413	0.165	0.264	0.102	1.013

Table 2 Buckling analysis results

Subshell	$\bar{F}_{cr}$	n at $\bar{F}_{cr}$
I	1.33	15
II	2.25	15
III	2.66	15

Table 3 Influence of number of stringers on buckling load of SSII

Number of stringers, M	$\bar{F}_{cr}$
30	2.15
40	2.60
50	2.91
60	3.16

Table 4 Influence of spacing on buckling load

Dimensionless ring spacing, l/R	$\bar{F}_{cr}$
0.50	3.23
0.75	2.43
1.00	1.92
2.00	1.33

50% of each parameter, thus suggesting that stiffener eccentricity need not be considered. However, this is definitely not so, because if the stiffeners are placed on the outside rather than on the inside of the cylinder, the dynamic buckling load is increased by 28.2%.

The number of stiffeners, rather than the stiffener cross-sectional area, is the important design consideration, which is not surprising, in view of the local buckling behavior observed previously. The stiffeners can exert only a torsional restraint in the local buckling mode, so that "beefing up" the stiffeners affects the buckling load only slightly. An optimal aircraft fuselage design is suggested to be one in which the stiffener cross section is reduced until general instability is on the verge of predominating over local buckling. Then the reduction in stiffener weight could be used to increase the total number of stiffeners, resulting in a more crashworthy aircraft with no increase in weight.

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A Criterion for Assessing Wind-Tunnel Wall Interference at Mach 1

Theodore R. Goodman*
Oceanics, Inc., Plainview, N. Y.

Introduction

IN 1958 Spreiter and Alksne introduced the concept of local linearization to calculate pressures on airfoils for Mach numbers near one.[1] They subsequently extended the concept to bodies of revolution (or, for slender configurations, to equivalent bodies of revolution). A comprehensive review has been presented by Spreiter.[2] It will be shown how the same ideas can be used to determine a criterion for assessing wind tunnel wall interference. Using the criterion, it becomes possible, with the aid of the transonic similarity laws, to design transonic wind tunnel experiments that are virtually interference-free.

The Two-Dimensional Case

The flow is assumed to be inviscid and irrotational, and the free stream Mach number is unity. In that case, a perturbation velocity potential ϕ exists such that the axial perturbation velocity $u = \partial\phi/\partial x$, while the vertical perturbation velocity $w = \partial\phi/\partial z$. This velocity potential obeys the nonlinear partial differential equation

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{\gamma + 1}{U_\infty} \frac{\partial \phi}{\partial x} \frac{\partial^2 \phi}{\partial x^2} \tag{1}$$

where x is the streamwise direction and z is normal to it.

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*Vice-President. Associate Fellow AIAA.